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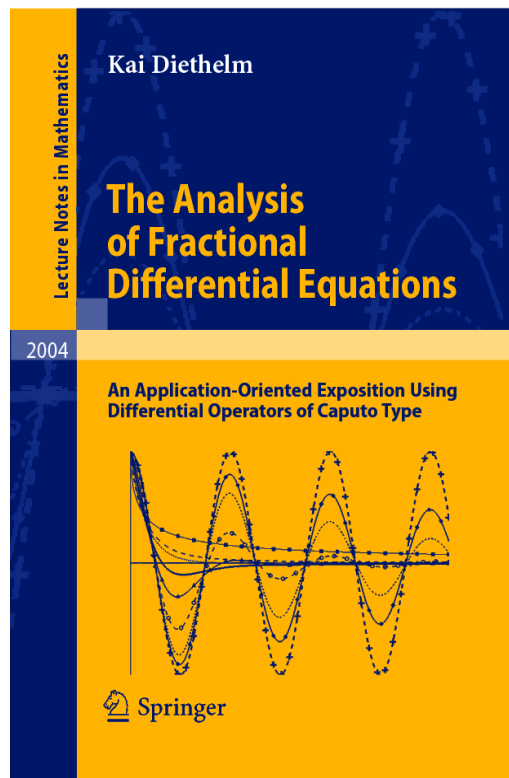
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## BOOK ANNOUNCEMENT: THE ANALYSIS OF FRACTIONAL DIFFERENTIAL EQUATIONS

An Application-Oriented Exposition Using  
Differential Operators of Caputo Type

by K. Diethelm



### Abstract

Among the many possible definitions of differential operators of fractional (i.e., non-integer) order, the one usually attributed to Caputo is particularly important because of its outstanding significance for the modelling of many memory-dependent phenomena in physics, engineering, finance, and various other areas. In this book we give a comprehensive account of these differential operators and the corresponding integral operators. Based on these results we then develop a thorough analysis of the differential equations with Caputo-type derivatives. This includes, among others, results concerning the existence and uniqueness of solutions, the dependence of the solutions on the given data, and the long-term behaviour of the solutions. Moreover we briefly compare the properties of Caputo-type equations to those of Riemann-Liouville type for which a detailed theory has been known for some time now but which are mainly of interest for theoretical, purely mathematical reasons and not from the point of view of applications. Our goal in this book is to provide the results derived in a way that is mathematically precise and rigorous and, at the same time, readable also for researchers from outside of mathematics who need to use Caputo differential equations in their work. We also believe that the theory developed in this book can serve as a solid foundation for a theoretical analysis and thorough understanding of methods for the numerical solution of these equations.

*MSC 2010:* 34A08, 34A12, 34-02, 34-01, 26A33, 33E12

*Key Words and Phrases:* fractional differential equation; fractional derivative of Caputo type; existence, uniqueness and stability of solutions; single- and multi-term differential equations; Mittag-Leffler functions

### Cover text

Fractional calculus has been developed by pure mathematicians since the middle of the 19th century. Some 100 years later, engineers and physicists have found applications for these concepts in their areas. However there has traditionally been little interaction between these two communities. In particular, typical mathematical works provide deep results about aspects with rather little significance in applications, and the engineering literature often lacks mathematical detail and precision. This book bridges the gap between the two communities. It concentrates on the class of fractional derivatives most important in applications, the Caputo operators, and provides a self-contained, thorough and mathematically rigorous study of their properties

and of the corresponding differential equations. The text is a useful tool for mathematicians and researchers from the applied sciences alike. It can also be used as a basis for teaching a graduate course on fractional differential equations.

### Bibliographical data

**Title:** The Analysis of Fractional Differential Equations

**Subtitle:** An Application-Oriented Exposition Using Differential Operators of Caputo Type

**Author:** Kai Diethelm (GNS Gesellschaft für numerische Simulation mbH, Braunschweig, Germany, and Institut Computational Mathematics, Technische Universität Braunschweig)

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### Key Features of the Book

- The book provides a detailed mathematical description of the class of fractional differential operators that is most important in applications in physics, engineering, etc.
- It bridges the gap between aspects from pure mathematics and application-oriented questions.
- It contains a solid mathematical foundation on which researchers from outside of mathematics can build their models.
- It is written in a style suitable for use as textbook.

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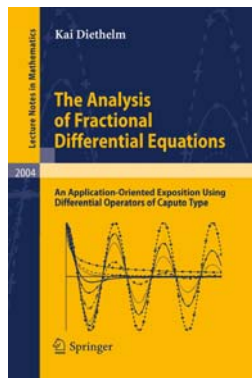
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## The Analysis of Fractional Differential Equations

An Application-Oriented Exposition Using Differential Operators of Caputo Type

Fractional calculus was first developed by pure mathematicians in the middle of the 19th century. Some 100 years later, engineers and physicists have found applications for these concepts in their areas. However there has traditionally been little interaction between these two communities. In particular, typical mathematical works provide extensive findings on aspects with comparatively little significance in applications, and the engineering literature often lacks mathematical detail and precision. This book bridges the gap between the two communities. It concentrates on the class of fractional derivatives most important in applications, the Caputo operators, and provides a self-contained, thorough and mathematically rigorous study of their properties and of the corresponding differential equations. The text is a useful tool for mathematicians and researchers from the applied sciences alike. It can also be used as a basis for teaching graduate courses on fractional differential equations.... [more on http://springer.com/978-3-642-14573-5](http://springer.com/978-3-642-14573-5)

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